QUANTUM COMPUTATION WITH "HOT" TRAPPED IONS

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We describe two methods that have been proposed to circumvent the problem of heating by external electromagnetic fields in ion trap quantum computers. Firstly the higher order modes of ion oscillation (i. e. modes other than the center-of-mass mode) have much slower heating rates, and can therefore be employed as a reliable quantum information bus. Secondly we discuss a recently proposed method combining adiabatic passage and a number-state dependent phase shift which allows quantum gates to be performed using the center-of-mass mode as the information bus, regardless of its initial state. *

INTRODUCTION

Ion trap quantum computers, first proposed by Ignacio Cirac and Peter Zoller of the University of Innsbruck[1], and demonstrated experimentally shortly afterwards by Dave Wineland and his collaborators at NIST Boulder[2], are, arguably, the most promising quantum computation technology for realizing systems of dozens of qubits in the foreseeable future. An ion trap quantum computer consists of a linear array of ions confined in a radio-frequency trap. Two internal states of the ion compose each qubit and the center-of-mass (CM) vibrational mode of the ions' collective oscillations acts as a quantum information bus, by means of which quantum logic gate operations can be performed between pairs of ions. The computer is controlled by a series of appropriately tuned laser pulses, which can change the internal states of each ion and, at the same time, exciting or destroying a quantum of the ions' collective oscillations.

The most daunting problem to be overcome in the development of this technology is the very fragile nature of the quantum mechanical ground-state of the CM mode. Cooling and maintaining the ions in this ground state are required for performing quantum logic gates in the manner proposed by Cirac and Zoller. Any excitation by external fields will diminish the accuracy of logic gates, thus leading to unreliable performance of the quantum computer as a whole, and maybe making the implementation of even simple algorithms impossible.

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Elimination of all possible causes of heating is an incredibly demanding task. Thus it is desirable to investigate methods for performing quantum logic gates without the necessity of being in the ground state of the CM mode. In this paper, we will briefly describe two possible methods by which this problem can be circumvented (fuller accounts can be found elsewhere[3,4]). Other schemes for tackling this problem have also been investigated by different authors[5,6].

SUPRESSION OF HEATING OF 'HIGHER' OSCILLATION MODES

As mentioned above, ion trap quantum computers transfer quantum information between different qubits by means of quanta of the ions' collective oscillation modes. Because of the Coulomb interaction between the different ions, the oscillations of a string of ions will be strongly coupled, and is best considered in terms of *normal modes.* Of these modes, which have been described in detail elsewhere[7], the most convenient mode to use for quantum computation is the CM mode, in which all of the ions oscillate as if they are rigidly clamped together. This mode has two advantages: it is well resolved in frequency space and all of the ions couple to it with equal strength. However, the transfer of information from one ion to another relies on these oscillatory modes being carefully prepared in their quantum ground states. If this were not the case, and the ions were oscillating randomly, quantum logic could not be performed reliably using Cirac and Zoller's scheme. The degradation of these modes due to various influences has been a subject of considerable study[3,8].

The different oscillatory modes are numbered $p = 1, 2, \ldots N$ in order of increasing oscillation frequency, N being the total number of ions in the string. The heating time (i. e. the time for the phonon excitation number to increase by one) for the p-th mode, $\tau_{N,p}$, is given by[3]:

$$\frac{\tau_1}{\tau_{N,p}} = \frac{\sum_{m,n=1}^{N} b_m^{(p)} b_n^{(p)} \gamma_{mn}}{\sqrt{\mu_p}}, \tag{1}$$

where τ_1 is the heating time for a single ion, $b_n^{(p)}$ is the normalized eigenvector and μ_p the eigenvalue, both of the p-th mode[7], and γ_{mn} is the degree of cross-coherence of the external fields acting on the n-th and m-th ions. The relative heating rates, calculated using eq. (1), when degree of cross-coherence is assumed to be a exponential function (i. e. $\gamma_{mn} = \exp\left[-|x_n - x_m|/\ell_{coh}\right]$, x_n being the position of the n-th ion and ℓ_{coh} the coherence length) are shown in fig. 1, for the case $N = 5$. One can easily calculate the heating time of the p-th mode in the limit of a coherent field ($\gamma_{mn} = 1$) and the limit of incoherent fields ($\gamma_{mn} = \delta_{n,m}$, where $\delta_{n,m}$ is the Kronecker delta):

$$\frac{\tau_1}{\tau_{N,p}} = \begin{cases} N & p = 1 \\ 0 & p > 1. \end{cases} \quad \text{coherent} \tag{2}$$

$$\frac{\tau_1}{\tau_{N,p}} = \frac{1}{\sqrt{\mu_p}} \quad \text{incoherent.} \tag{3}$$

The heating times in the coherent limit is a very important result: Only the lowest ($p = 1$) CM mode will be heated up by spatially coherent fields, the modes other than the CM mode remaining in their ground states. The separations of the ions is of the order of a few microns, while the wavelength of the external exciting fields (in resonance with the ions oscillation frequency) is hundreds of meters; thus the coherent case is the experimentally important case. This relatively rapid heating of the CM mode compared to the other modes has also been demonstrated experimentally[9].

Figure 1. Relative heating rates of different oscillation modes for $L = 5$ ions. For heating fields with long coherence lengths, the $p = 1$ center of mass mode is heated strongly, while the heating rates of the 'higher' modes is significantly reduced.

Drawbacks of this scheme are: the strength of the coupling of each ion to those higher order modes varies from ion to ion, making it harder to adjust the pulse durations; the higher modes are closer together in frequency space, thus making them harder to resolve; and also there is a 'Debye-Waller' factor due to heating of the CM mode which alters the strength of the laser interaction by an undetermined factor.

ADIABATIC PASSAGE AND THE D'HELON-MILBURN PHASE SHIFT

We will now discuss an entirely different approach to the problem of performing quantum logic with 'hot' trapped ions. We make use of the fact that although the ions are not necessarily in the CM mode ground state, they all share the same CM mode, thus enabling them to interact with each other. Our scheme performs a controlled-rotation (CROT) gate between a pair of ions designated control (c) and target (t). This gate operation consists of a conditional sign change which takes place only if both ions are in the excited state. It can be realized by a sequence of four laser pulses, illustrated symbolically in fig. 2.

We will assume that the CM phonon mode is in a pure state† given by the following formula:

$$|\phi\rangle = \sum_n a_n |n\rangle \ , \tag{4}$$

where a_n are a set of unknown complex coefficients and $|n\rangle$ is the Fock state of occupation number n. It will be convenient in what follows to introduce the odd and even

†The gate can be shown to work when the CM mode is in a mixed state: we assume a pure state for simplicity.

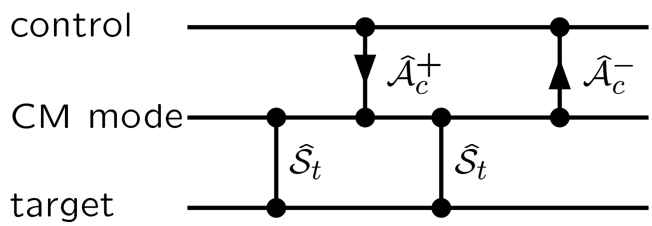

Figure 2. Schematic illustration of the steps involved in the CROT gate with hot ions. The individual steps are discussed in detail in the text.

parts of this wavefunction, viz. :

$$\begin{aligned} |+\rangle &= \sum_n a_{2n}|2n\rangle \ , \\ |-\rangle &= \sum_n a_{2n+1}|2n+1\rangle \ . \end{aligned} \tag{5}$$

We will also use the following notation for phonon states to which a single quantum has been added:

$$\begin{aligned} |-'\rangle &= \sum_n a_{2n}|2n+1\rangle \ , \\ |+'\rangle &= \sum_n a_{2n+1}|2n+2\rangle \ . \end{aligned} \tag{6}$$

The first step is a conditional phase change between odd phonon number states and the excited internal state of an ion, which can be carried out using an effect first considered by D'Helon and Milburn[11]. They introduced a Hamiltonian for a two-level ion at the node of a detuned standing wave laser field. In the limit of large detuning and for interaction times much greater than the vibrational period of the trap, this Hamiltonian for the jth ion is

$$\hat{H}^{(j)} = \hbar \hat{a}^\dagger \hat{a} \chi (\hat{\sigma}_z^{(j)} + 1/2) \ , \tag{7}$$

where $\hat{\sigma}_z^{(j)}$ is the population inversion operator for the jth ion, $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the CM mode, and $\chi = \eta^2\Omega^2/(N\delta)$ (η is the Lamb-Dicke parameter, Ω is the Rabi frequency for the transitions between the two internal states of the ions and δ the laser detuning). If we choose the duration τ of this interaction to be $\tau = \pi/\chi$, the time evolution is represented by the operator

$$\hat{\mathcal{S}}_j = \exp[-i\hat{a}^\dagger \hat{a}(\hat{\sigma}_z^{(j)} + 1/2)\pi] \ . \tag{8}$$

This time evolution flips the phase of the ion when the CM mode is in an odd state and the ion is in its excited state, thus providing us with a conditional phase shift for an ion and the CM mode.

The adiabatic passage[10] which we require for the next step of our gate operation can be realized as follows. We use two lasers, the pump (which couples the qubit state $|1\rangle_c$ to some second auxiliary state $|3\rangle_c$ and is detuned by an amount Δ) and the Stokes (which couples to the red side band transition $|2\rangle_c|n+1\rangle \leftrightarrow |3\rangle_c|n\rangle$, with the same detuning Δ) (see fig. 3). If the population we want to transfer adiabatically is initially in the state $|1\rangle_c|n\rangle$, we turn on the Stokes field and then slowly turn on the pump field until both lasers are turned on fully. Then we slowly turn off the Stokes laser: this is the famous "counter-intuitive" pulse sequence used in adiabatic passage[10]. The

Figure 3. Schematic illustration of the level scheme of the control ion used to realize the adiabatic passage operations $\hat{\mathcal{A}}_c^+$ and $\hat{\mathcal{A}}_c^-$.

adiabatic passage has to be performed very slowly. The condition in our scheme is that $T \gg 1/\Omega_{p,n}, 1/\Omega_{S,n}$, where T is the duration of the adiabatic passage and $\Omega_{p,n}$ ($\Omega_{S,n}$) are the effective Rabi frequencies for the pump and the Stokes transition, respectively. Using the adiabatic passage we can transfer the population from $|1\rangle_c|n\rangle$ to $|2\rangle_c|n+1\rangle$. To invert the adiabatic passage, we just have to interchange the roles of the pump and the Stokes field. We will denote the adiabatic passage by operators $\hat{\mathcal{A}}_j^+$ and $\hat{\mathcal{A}}_j^-$ defined as follows:

$$\begin{aligned} \hat{\mathcal{A}}_j^+ &: |1\rangle_j|n\rangle \to |2\rangle_j|n+1\rangle \\ \hat{\mathcal{A}}_j^- &: |2\rangle_j|n+1\rangle \to |1\rangle_j|n\rangle \ . \end{aligned} \tag{9}$$

Putting these two operations together in the sequence shown in fig. 2, we can write down the step-by-step states for our gate:

$$\begin{aligned}
&|00\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{S}_t} |00\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{\mathcal{A}}_c^+} |00\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{S}_t} \\
&\qquad\qquad\qquad |00\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{\mathcal{A}}_c^-} |00\rangle(|+\rangle+|-\rangle) \\
&|01\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{S}_t} |01\rangle(|+\rangle-|-\rangle) \xrightarrow{\hat{\mathcal{A}}_c^+} |01\rangle(|+\rangle-|-\rangle) \xrightarrow{\hat{S}_t} \\
&\qquad\qquad\qquad |01\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{\mathcal{A}}_c^-} |01\rangle(|+\rangle+|-\rangle) \\
&|10\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{S}_t} |10\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{\mathcal{A}}_c^+} |20\rangle(|+'\rangle+|-'\rangle) \xrightarrow{\hat{S}_t} \\
&\qquad\qquad\qquad |20\rangle(|+'\rangle+|-'\rangle) \xrightarrow{\hat{\mathcal{A}}_c^-} |10\rangle(|+\rangle+|-\rangle) \\
&|11\rangle(|+\rangle+|-\rangle) \xrightarrow{\hat{S}_t} |11\rangle(|+\rangle-|-\rangle) \xrightarrow{\hat{\mathcal{A}}_c^+} |21\rangle(|-'\rangle-|+'\rangle) \xrightarrow{\hat{S}_t} \\
&\qquad\qquad\qquad |21\rangle(-|-'\rangle-|+'\rangle) \xrightarrow{\hat{\mathcal{A}}_c^-} -|11\rangle(|+\rangle+|-\rangle).
\end{aligned} \tag{10}$$

Thus we end up with a controlled rotation gate between the ions c and t. A controlled-NOT (CNOT) gate can be realized by performing $\pi/2$ rotation pulses on the target qubit both before and after this series of operations.

A possible source of error in performing gate operations using this scheme is the heating during gate operations. To perform logic operations, effectively the quantum information stored in the two levels of the control qubit is transferred to the even and odd states of the CM mode. Heating mixes these two states, thereby degrading the information stored. Other experimental issues are the phase change induced during the adiabatic passages: the two operations $\hat{\mathcal{A}}_c^+$ and $\hat{\mathcal{A}}_c^-$ together result in a phase shift for the state $|1\rangle_c$; also the Stokes and pump fields cause A. C. Stark shifts to other levels, resulting in phase changes. These effects can be compensated by judicious choices of free parameters such as the time taken for the adiabatic passage or the detuning. The D'Helon- Milburn conditional phase operator $\hat{\mathcal{S}}_t$ relies on the assumption of the Lamb-Dicke regime as well as a standing wave field acting on target ion. However, standing waves are difficult to achieve in practice, but it may be be possible to obtain similar effects using traveling waves; further for the low trap frequencies required to resolve spatially individual ions with a focused laser, the Lamb-Dicke regime may not be valid. All of these questions, as well as the study of heating and decoherence mechanisms will be the subject of future papers.

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